1

Fourier Transformation

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

$1 \circ\!\!-\!\!\bullet\ \delta(f)$ $\quad e^{j2\pi f_0 t} \circ\!\!-\!\!\bullet\ \delta(f-f_0)$

$\delta(t) \circ\!\!-\!\!\bullet\ 1$ $\quad \delta(t-t_0) \circ\!\!-\!\!\bullet\ e^{-j2\pi f t_0}$

$\cos(2\pi f_0 t) \circ\!\!-\!\!\bullet\ \frac{1}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right)$

$\sin(2\pi f_0 t) \circ\!\!-\!\!\bullet\ \frac{1}{2j}\left(\delta(f-f_0) - \delta(f+f_0)\right)$

$\mathrm{rect}(t/T) \circ\!\!-\!\!\bullet\ T\,\mathrm{si}(\pi f T)$ $\quad 1/(\pi t) \circ\!\!-\!\!\bullet\ -j\,\mathrm{sign}(f)$

$\mathrm{si}(\pi t/T) \circ\!\!-\!\!\bullet\ T\,\mathrm{rect}(fT)$

Mittlere Leistung

$$S_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

Energie des Signals

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

Pegelrechnung

$$10 \cdot \log_{10} SNR = 10 \log_{10}\left(\frac{S}{N}\right) = 20 \log_{10}\left(\frac{S_{eff}}{N_{eff}}\right)$$

Limitationsstörung:

$$N_L = 2\int_1^{\infty} \varepsilon^2 f_q(\varepsilon+1)\, d\varepsilon = 2 \cdot \int_1^{\infty} (\varepsilon-1)^2 f_q(\varepsilon)\, d\varepsilon$$

$$SNR_L = S_q / N_L = q_{eff}^2 / N_L$$

Einheiten

$T_B\ [s]$

$R_T = \frac{1}{T_B} \left[\frac{bit}{s}\right]$

$\Gamma_q = \frac{B_{NF}}{B_{HF}}$ $\quad J = \frac{1}{\Gamma_q}$

$\underbrace{\Gamma_q}_{\text{Bandbreiteneff.}}$ $\quad \underbrace{J}_{\text{Banderweiterung}}$

$\Gamma_A = \frac{R_T}{B_{HF}} \left[\frac{bit/s}{Hz}\right]$

$\varphi_c \hat{=}$ Trägerphase

$M_q \hat{=}$ Amplitudenstufen

$\Delta q \hat{=}$ Breite Quantisierungsintervall

$C \left[\frac{bit}{\text{Kanalnutzung}}\right]$

$R \left[\frac{bit}{Sym}\right]$

$R_c \left[\frac{bit}{\text{Codesym}}\right]$

Ausbreitung isotroper Strahler:

Effective isot. radiated power:

$EIRP = S_{Tx} \cdot G_S$ $\quad G_S \hat{=}$ Antennengewinn beim Sender

Leistungsdichte mit Abstand d:

$$\frac{EIRP}{4\pi d^2} \left[\frac{W}{m^2}\right]$$

Empfangene Leistung: $S(d) = EIRP \cdot \frac{A_e}{4\pi d^2}\ [W]$

$$G_E = \frac{4\pi}{\lambda^2} A_e$$

$$S_{Rx}(d) = S_{Tx}\, G_S\, G_E \frac{1}{(4\pi)^2} \left(\frac{\lambda}{d}\right)^2$$

Übertragungsfaktor: $D = \sqrt{S_{Rx}(d)/S_{Tx}}$

Signaldämpfung: $= -20 \log_{10}(D)$

Effektive Wirkfläche Parabol:

$$A_e = \frac{\pi\, d^2}{4} \cdot \eta \qquad \eta \hat{=} \text{Beleuchtungsgrad}$$

Hornantenne:

$$A_e = A \cdot \eta$$

isotrope Absorber:

$$A_{is} = \frac{\lambda^2}{4\pi}$$

Rauschen:

$\Phi_{nHF,nHF}(f) = \frac{N_0}{2} \leftarrow$ LDS

$\varphi_{nHF,nHF}(\tau) = \frac{N_0}{2} \delta(\tau) \leftarrow$ AKF

$\frac{N_0}{2} \hat{=}$ zweiseitige Rauschleistungsdichte

Übertragungsfkt Kabel:

$$H_K(f,d) = 10^{-d \cdot a_{10}(f)/20}$$

AM o. Tr.

$$s_{HF}(t) = q(t) \cdot z(t)$$

$$= q(t) \cdot \sqrt{2}\, z_{eff} \cdot \cos(2\pi f_c t + \varphi_c)$$

$$S_{Tx} = z_{eff}^2 \cdot S_q = z_{eff}^2 \cdot q_{eff}^2$$

ECB Sig.

$s(t) = z_{eff} \cdot q(t) \cdot e^{j\varphi_c}$

$S(f) = z_{eff} \cdot Q(f) \cdot e^{j\varphi_c}$

ECB-Signal

$\longrightarrow$ ECB Trafo

$$x(t) = \frac{1}{\sqrt{2}} x_{HF}^{+}(t) \cdot e^{-j2\pi f_0 t}$$

$$= \frac{1}{\sqrt{2}} \left(x_{HF}(t) + j\,\mathcal{H}\{x_{HF}(t)\}\right) \cdot e^{-j\pi f_0 t}$$

$$X(f) = \frac{1}{\sqrt{2}} X_{HF}^{+}(f+f_0) =$$

$$\frac{1}{\sqrt{2}} \left(1 + \mathrm{sign}(f+f_0)\right) \cdot X_{HF}(f+f_0)$$

ECB $\to$ Rücktrafo

$$x_{HF}(t) = \sqrt{2}\, \mathrm{Re}\{x(t)\, e^{+j2\pi f_0 t}\}$$

$$= \frac{\sqrt{2}}{2} \left(x(t) \cdot e^{j2\pi f_0 t} + x^*(t) \cdot e^{-j2\pi f_0 t}\right)$$

$$X_{HF}(f) = \frac{\sqrt{2}}{2} \left(X(f-f_0) + X^*(-(f+f_0))\right)$$

AM m. Tr.

$$s_{HF}(t) = \hat{s}\,(1 + m \cdot q(t)) \cdot \cos(2\pi f_c t + \varphi_c)$$

$$\hat{s} = \sqrt{2}\, z_{eff} \cdot q_= \qquad m = \frac{\sqrt{2} \cdot z_{eff}}{\hat{s}} = \frac{1}{q_=}$$

$$S_{Tx} = \frac{1}{2} \hat{s}^2 \cdot (1 + m^2 q_{eff}^2)$$

ECB-Sig.

$s(t) = \frac{1}{\sqrt{2}} \hat{s}\,(1 + m \cdot q(t)) \cdot e^{j\varphi_c}$

$S(f) = \frac{1}{\sqrt{2}} \hat{s}\,(\delta(f) + m \cdot Q(f)) \cdot e^{j\varphi_c}$

Wirkungsgrad

$$\eta = \frac{m^2 q_{eff}^2}{1 + m^2 q_{eff}^2}$$

QAM

z_{eff} für Mod / z_{eff} für Demod

$$s_{HF}(t) = q_I(t) \cdot \sqrt{2}\, z_{eff} \cdot \cos(2\pi f_c t) - q_Q(t) \cdot \sqrt{2}\, z_{eff} \cdot \sin(2\pi f_c t)$$

$$= \sqrt{2} \cdot z_{eff} \cdot \left(q_I(t) \cdot \cos(2\pi f_c t) - q_Q(t) \cdot \sin(2\pi f_c t)\right)$$

$$S_{HF}(f) = \frac{1}{\sqrt{2}} \cdot z_{eff} \cdot \left[Q_I(f-f_c) + Q_I(f+f_c) + j\left(Q_Q(f-f_c) - Q_Q(f+f_c)\right)\right]$$

ECB-Sig. $s(t) = z_{eff} \cdot (q_I(t) + j\, q_Q(t))$

$S(f) = z_{eff} \cdot (Q_I(f) + j\, Q_Q(f))$

Demod: $v(t) = q(t) \cdot e^{-j(2\pi \Delta f t + \Delta\varphi_c)}$

Fehler $S_{Tx} = z_{eff}^2 (q_{eff_I}^2 + q_{eff_Q}^2)$

Störabstände

$\boxed{EM}$ ist QAM mit $q_Q(t) = \pm \mathcal{H}\{q_I(t)\}$: $+ \hat{=}$ Regellage, $- \hat{=}$ Kehrlage

$S_{Tx} = 2 \cdot z_{eff}^2 \cdot q_{eff}^2$

Verf	SNR_{NF}
AM o. Tr	SNR_0
AM m. Tr	$SNR_0 \cdot \eta$
QAM	SNR_0
EM/RM	SNR_0

$SNR_{HF} = \frac{S_{Rx}}{B_{HF} \cdot N_0}$ $SNR_{NF} = \frac{S_V}{N_{K}}$ Leistung des Sinkensignals / Kanalstörung

$10 \log_{10}(SNR_{HF})$ [dB] $10 \log_{10}(SNR_{NF})$ [dB]

AWGN-Modell: $S_{Tx} = S_{Rx}$

$\boxed{\text{Vergleichsstörabstand}}$

$10 \log_{10}(SNR_0) = 10 \log_{10}\left(\frac{S_{Rx}}{B_{NF} \cdot N_0}\right)$ [dB]

$10 \log_{10}(SNR_0) = 10 \log_{10}(SNR_{HF}) - 10 \log_{10}(\Gamma_a)$

$\boxed{FM}$ $\Delta f \hat{=}$ Frequenzhub

$S_{Tx} = s_{eff}^2 = \hat{s}^2/2$

Quellensignal: $q(t) = \cos(2\pi f_q t + \varphi_q)$

$\boxed{\text{Phasenhub}}$

$\Delta\varphi = \frac{\Delta f}{f_q}$

$\boxed{\text{Sendesignal}}$

$s(t) = s_{eff} \sum_{n=-\infty}^{\infty} J_n(\Delta\varphi) \cdot e^{j2\pi n f_q t}$

$S(f) = s_{eff} \sum_{n=-\infty}^{\infty} J_n(\Delta\varphi)\, \delta(f - n f_q)$

$S_{HF}(f) = \frac{1}{\sqrt{2}}[S(f-f_c) + S(f+f_c)]$

$\boxed{\text{Carson-Formel}}$

$B_{HF} \approx 2(\Delta f + \gamma B_{NF})$ $\gamma \in \{1$ mittlere Qualität 98%; 2 hohe Qualität 99%$\}$

$J = 2(\Delta f / B_{NF} + \gamma)$

$\boxed{\text{Demod}}$

$\boxed{\arg\{\cdot\}} \to \boxed{\frac{d}{dt}} \to \boxed{\frac{(\cdot + \pi)\bmod(2\pi)}{2\pi\Delta f}} - \boxed{h_{TP}}$

$SNR_{NF} = 3 q_{eff}^2 \left(\frac{\Delta f}{B_{NF}}\right)^2 \cdot SNR_0$

$SNR_{NF} = 3 q_{eff}^2 \left(\frac{J}{2} - \gamma\right)^2 \cdot SNR_0$

$\boxed{\text{Gewinn durch Pre/Deemphase}}$

$\frac{SNR_p}{S/N} = \frac{S \cdot N}{\left(\int_{-B_{NF}}^{B_{NF}} \sqrt{\Phi_{qq}(f)\, \Phi_{nn}(f)}\, df\right)^2}$

$\boxed{PCM}$ $R_T = \frac{1}{T_b} = f_A \lceil \log_2(M_q) \rceil = f_A \cdot n \left[\frac{Bit}{s}\right]$

Quantisierungsgeräusch im Intervall „i"

$N_{Q_i} \approx \frac{\Delta q_i^2}{12}$ $\boxed{\text{Mittlere Leistung}}$

$N_Q = \sum_{i=1}^{M_q} f_q(r_i) \cdot \frac{\Delta q_i^3}{12}$

$N_{Q_c} = N_Q \frac{2 \cdot B_{NF}}{f_A}$

$\boxed{\text{Störabstand gleichm. Quant.}}$

$SNR_{NF} = \frac{q_{eff}^2}{N_Q} = 3 q_{eff}^2 \cdot M_q^2$

$10 \log_{10}(SNR_{NF}) = 4{,}77\,dB + 10 \log_{10}(q_{eff}^2) + n \cdot 6{,}02\,dB$

$\boxed{\text{Gesamte Störung}}$ $SNR_V = \frac{q_{eff}^2}{N_\varepsilon + N_{Q_c} + N_L}$ (Bitfehler, Quant., Übersteuerung)

$\boxed{\text{Störung durch Bitfehler}}$

$N_\varepsilon =$ ~~[illegible]~~ normaler Dualcode:

$N_\varepsilon \approx BER \cdot \frac{4}{3}$ für $n \geq 3$

symm. Dualcode

$N_\varepsilon \approx (4 q_{eff}^2 + \frac{1}{3}) \cdot BER$

$\boxed{\text{Kompandierung}}$

$N_Q \approx \frac{1}{3 M_q^2} \cdot \int_{-1}^{1} \frac{f_q(q)}{(u_k'(q))^2}\, dq$ $\Delta q_i \approx \frac{\Delta x}{u_k'(q)}$

$\boxed{\text{optimale Kompressorkennlinie}}$ $u_k'(q) = c \cdot \sqrt[3]{f_q(q)}$

$N_Q \approx \frac{1}{3 M_q^2} \cdot 2 \cdot \left(\int_0^1 \sqrt[3]{f_q(q)}\, dq\right)^3$ $\Delta q_i \approx \frac{2/M_q}{u_k'(q)}$

$\Pr(q \in I_i) \to \int_{I_i} f_q(q)\, dq \approx f_q(q) \cdot \Delta q_i$

$\boxed{\text{Gewinn durch Kompandierung:}}$ Laplace-Vert: $10 \log_{10}(SNR_{NF}) = n \cdot 6{,}02\,dB - 6{,}53\,dB$

Gaußvert: $10 \log_{10}(SNR_{NF}) = n \cdot 6{,}02\,dB - 4{,}35\,dB$

$\boxed{\text{A-Law}}$ $A = \frac{1}{q_c}$ $q_c \hat{=}$ linearer Schwellwert

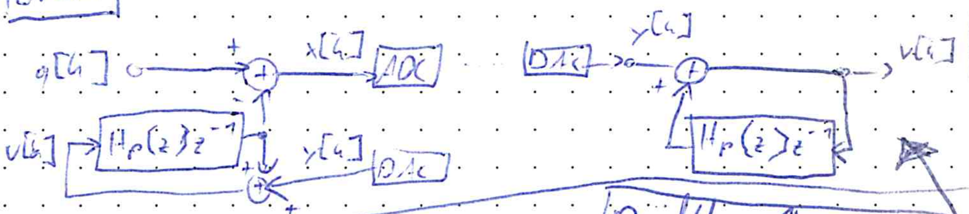
$u_k(q) = \begin{cases} q \cdot \frac{A}{1 + \ln(A)} & 0 \leq q \leq q_c \\ \frac{1 + \ln(Aq)}{1 + \ln(A)} & q_c \leq q \leq 1 \end{cases}$ $u_k(-q) \hat{=} -u_k(q)$

$SNR_{NF} = \frac{q_{eff}^2}{N_Q} = 3 M_q^2 \frac{a^2}{A^2} = 3 M_q^2 \frac{1}{(1 + \ln(A))^2}$

$10 \log_{10}(SNR_{NF}) = 10 \log_{10}(3 M_q^2) - 20 \log_{10}(1 + \ln(A))$

$\sigma^2_{x,min} \hat{=}$ minimale Varianz

$\boxed{\text{DPCM}}$



$\boxed{\text{Prädiktor 1ter Ordn.:}}$

$$h_p[0] = \frac{\phi_{qq}[1]}{\phi_{qq}[0]}$$

$$\sigma^2_{x,min} = \frac{\phi^2_{qq}[0] - \phi^2_{qq}[1]}{\phi_{qq}[0]}$$

$$G_{p,opt} = -10\log_{10}\left(1 - \left(\frac{\phi_{qq}[1]}{\phi_{qq}[0]}\right)^2\right)$$

$$X(z) = Q(z) - H_p(z) z^{-1} = (1 - H_p(z)z^{-1}) \cdot Q(z) = F(z) \cdot Q(z)$$

$$G_{p,opt} = -10\log_{10}\left(1 - \frac{\phi^2_{qq}[1] + \phi^2_{qq}[2] - \frac{2\phi^2_{qq}[1]\phi_{qq}[2]}{\phi_{qq}[0]}}{\phi^2_{qq}[0] - \phi^2_{qq}[1]}\right)$$

$\boxed{\text{Prädiktor 2ter Ordnung: (Cramersche Regel)}}$

$$h_p[0] = \frac{\phi_{qq}[0]\phi_{qq}[1] - \phi_{qq}[1]\phi_{qq}[2]}{\phi^2_{qq}[0] - \phi^2_{qq}[1]}$$

$$h_p[1] = \frac{\phi_{qq}[0]\phi_{qq}[2] - \phi^2_{qq}[1]}{\phi^2_{qq}[0] - \phi^2_{qq}[1]}$$

$\boxed{\text{Yule-Walker-Gleichungen}}$ (Grad „p")

$$\begin{bmatrix} \phi_{qq}[0] & \phi_{qq}[1] & \cdots & \phi_{qq}[p] \\ \phi_{qq}[1] & \ddots & & \\ \vdots & & \ddots & \phi_{qq}[1] \\ \phi_{qq}[p] & \cdots & \phi_{qq}[1] & \phi_{qq}[0] \end{bmatrix} \cdot \begin{bmatrix} h_p[0] \\ h_p[1] \\ \vdots \\ h_p[p] \end{bmatrix} = \begin{bmatrix} \phi_{qq}[1] \\ \phi_{qq}[2] \\ \vdots \\ \phi_{qq}[p+1] \end{bmatrix}$$

$$\sigma^2_{x,min} = \phi_{qq}[0] - \sum_{k=0}^{p} h_p[k]\phi_{qq}[k+1]$$

$$G_{p,opt} = -10\log\left(1 - \frac{\sum_{k=0}^{p} h_p[k]\phi_{qq}[k+1]}{\phi_{qq}[0]}\right)$$

$\boxed{\text{DTFT:}}$

$$\mathcal{F}_*\{x[k]\} = X(e^{j2\pi F}) = \sum_{-\infty}^{\infty} x[k] e^{-j2\pi Fk}$$

$\boxed{\text{Informationstheorie}}$

$M_X \hat{=}$ Anzahl Quellsymbole

Entropie: $H(X) = -\sum_{i=1}^{M_X} Pr\{x_i\} \cdot \log_2(Pr\{x_i\})$ $\left[\frac{bit}{sym}\right]$

$$e_2(p) = -p\,ld(p) - (1-p)\cdot ld(1-p)$$

$\boxed{\text{Huffman Codierung}}$

$L \hat{=}$ Codewortlänge

$n_i \hat{=}$ Codewortlänge

$p_i \hat{=}$ Wortwahrscheinlichkeit

Mittlere Codewortlänge: $\bar{n}_L = \sum_{i=1}^{M_X^L} n_i \cdot p_i$

Minimale mittlere Wortlänge: $\bar{n} = \frac{n_i}{L}$

Mittlere Zahl binärer Codesyms je Quellensym:

$$H(X) \leq \frac{\bar{n}_L}{L} < H(X) + \frac{1}{L} \quad \Rightarrow \text{lange Blöcke: } \lim_{L\to\infty} \frac{\bar{n}_L}{L} = H(X)$$

Wechselseitige Information:

$$H(XY) = -\sum_{i=1}^{M_X}\sum_{j=1}^{M_Y} Pr\{x_i y_j\}\, ld(Pr\{x_i y_j\})$$

$$H(X) \Leftrightarrow H(Y)$$

$$H(X|Y) = \sum_{j=1}^{M_Y} Pr\{y_j\} H(X|y_j)$$

$$H(X|y_j) = -\sum_{i=1}^{M_X} Pr\{x_i|y_j\}\, ld(Pr\{x_i|y_j\})$$

$$\Downarrow$$

$$H(X|Y) = -\sum_{i=1}^{M_X}\sum_{j=1}^{M_Y} Pr\{x_i, y_j\} \cdot ld(Pr\{x_i|y_j\})$$

$$Pr\{x_i, y_j\} = Pr\{y_j\} \cdot Pr\{x_i|y_j\}$$

$$H(XY) = H(X) + H(Y|X) = H(Y) + H(X|Y)$$

$$H(X|Y) = H(XY) - H(Y) \leq H(X)$$

$$I(X;Y) = \sum_{i=1}^{M_X}\sum_{j=1}^{M_Y} Pr\{x_i, y_j\} \cdot ld\left(\frac{Pr\{x_i|y_j\}}{Pr\{x_i\}}\right)$$

$$= \sum_{i=1}^{M_X}\sum_{j=1}^{M_Y} Pr\{x_i, y_j\} \cdot ld\left(\frac{Pr\{y_j|x_i\}}{Pr\{y_j\}}\right)$$

$I(X;Y)\left[\frac{bit}{K.Nutzung}\right]$

$$I(X;Y) = H(X) - H(X|Y) > 0 \qquad C = \max_{Pr\{X\}}$$

$\underline{BSC}$: $C = 1 - e_2(BER)$ $\quad \left\{\begin{array}{l}\underline{BEC} \\ C = 1 - ER\end{array}\right.$

$R \leq C$

$\boxed{\text{Coderate}}$ Mittlerer Infogehalt der Kanalsymbole:

$$R_c = \frac{k}{n}\, ld(M_c)$$

$\boxed{\text{Binärcode}}$ $M_c = 2$

k Infosymbole
n total
Rest = Redundanz

Kont. Transinformation:

$$I(X;Y) = -\int_{-\infty}^{\infty} f_Y(y)\, ld(f_Y(y))\,dy + \iint_{-\infty}^{\infty} f_{XY}(x,y)\, ld(f_{Y|X}(y))\,dx\,dy$$

$\boxed{\text{AWGN-Kapazität}}$ $\quad \frac{S}{N} = \frac{\sigma_x^2}{\sigma_N^2}$

$C_{AWGN} = \frac{1}{2} ld(1 + \frac{S}{N}) \left[\frac{bit}{K.Nutzung}\right]$ reel. kont

$C = ld(1 + \frac{S}{N}) \left[\frac{bit}{K.Nutzung}\right]$ kompl. disk

Differentielle Entropie: $h(X) = -\int_{-\infty}^{\infty} f_X(x)\, ld(f_X(x))\,dx$

bei gleichm. Quantisierung: $H(Y) \approx h(X) - ld(\Delta q)$

Transinfo bei Störung: $I(X;Y) = h(Y) - h(N) \left[\frac{bit}{K.Nutzung}\right]$

$\sigma_N^2 \hat{=}$ Varianz der Störung $\qquad$ Diff. Entrop. der Störung $h(N) = \frac{1}{2} ld(2\pi e \sigma_n^2)$

Austausch Rate / Effizienz

$R = R_T \cdot T = \frac{T}{T_b}$

$N = \frac{N_0}{T}$

$\frac{E_b}{N_0} = \frac{1}{R}(2^R - 1)$

$T \hat{=}$ Kanalbenutzungsintervall

$C_T = B_{HF} \cdot \mathrm{ld}\left(1 + \frac{S}{N}\right) \left[\frac{\text{bit}}{s}\right]$

$\Gamma_d = \frac{R_T}{B_{HF}} \left[\frac{\text{bit}/s}{Hz}\right]$

Shannon Grenze: $\frac{E_b}{N_0} = \frac{1}{\Gamma_d}(2^{\Gamma_d} - 1)$

Bandbegrenzter AWGN-Kanal

hoher Störabstand

$C_T \approx \frac{1}{3{,}0103} B_{HF} \cdot 10 \log_{10}(SNR_{HF})$

Digitale Übertragung analoger Signale

n bit pro Abtastung

$SNR_0 = \frac{E_b}{N_0} \cdot 2n$

$\Gamma_d = \Gamma_d(2n)$

$SNR_{NF} = (SNR_{HF} + 1)^J - 1 = \left(\frac{SNR_0}{J} + 1\right)^J$

Rate Distortion Bound (Gauss): $SNR = 2^{2n} - 1$

PAM

Digitales PAM Signal: $s(t) = \sum_{k=-\infty}^{\infty} a[k] g(t - kT)$

gleichzeitig das ECB Signal

$R = R_c \cdot \mathrm{ld}(M)$

$s_{HF}(t) = \sqrt{2}\, \mathrm{Re}\{s(t) \cdot e^{j2\pi f_c t}\}$

$s_{HF}(t) = \sqrt{2}\left(\sum_{k=-\infty}^{\infty} \mathrm{Re}\{a[k]\} g(t - kT)\right) \cdot \cos(2\pi f_c t)$

$-\sqrt{2}\left(\sum_{k=-\infty}^{\infty} \mathrm{Im}\{a[k]\} g(t - kT)\right) \cdot \sin(2\pi f_c t)$

Autokorrelation:

$\phi_{ss}(\tau) = \frac{1}{T} \sum_k \phi_{aa}[k] \cdot \varphi_{gg}(\tau - kT)$

LDS:

$\Phi_{ss}(f) = \frac{1}{T} \Phi_{aa}(e^{j2\pi fT}) |G(f)|^2$

Mittleres LDS:

$\bar{\Phi}_{ss}(f) = \sigma_a^2 \frac{|G(f)|^2}{T} + |m_a|^2 \frac{|G(f)|^2}{T^2} \sum_{\ell=-\infty}^{\infty} \delta\left(f - \frac{\ell}{T}\right)$

$m_a = 0$: $S_{Tx} = \frac{\sigma_a^2}{T} E_g$

Sendeleistung

mittl. Energie je Symbol: $E_s = S_{Tx} \cdot T = \sigma_a^2 \cdot E_g$

mittl. Energie je Infobit: $E_b = \frac{E_s}{R} = \frac{S_{Tx}}{T_b}$

ohne Kod.: $E_b = \frac{\sigma_a^2 E_g}{\mathrm{ld}(M)}$

ASK bipolare: $\sigma_a^2 = \frac{M^2 - 1}{3}$, $m_a = 0$

unipolare: $\sigma_a^2 = \frac{M^2 - 1}{3}$, $m_a = M - 1$

$d_{E,min} = 2$

PSK

$d_{E,min} = 2\sin(\pi/M)$

Fehlerwahrscheinlichkeiten

Bipolare ASK: $SER = \frac{2M - 2}{M} Q\left(\sqrt{d_{min}^2 \frac{E_b}{N_0}}\right)$

$SER \le N_{min} Q\left(\sqrt{d_{min}^2 \ldots}\right)$

$d_{min}^2 = d_{E,min}^2 \frac{E_g}{2E_b} = \frac{1}{2} d_{E,min}^2 \cdot R / \sigma_a^2$

$d_{E,min} \begin{cases} 2 & \text{ASK}, \ldots \\ 2\sin(\pi/M) & \ldots \end{cases}$

$N_{min} \hat{=}$ mittlere Zahl von nächsten Nachbarsignalen

$BER \approx \frac{N_{min}}{\mathrm{ld}(M)} Q\left(\sqrt{d_{min}^2 \frac{E_b}{N_0}}\right)$

M-ASK bipolar: $d_{min}^2 = \frac{6\, \mathrm{ld}(M)}{M^2 - 1}$

unipolar: $d_{min}^2 = \frac{6\, \mathrm{ld}(M)}{4M^2 - 6M + 2}$

M-PSK: $d_{min}^2 = 2\, \mathrm{ld}(M) \cdot \sin^2\left(\frac{\pi}{M}\right)$

M-QAM (quadratisch): $d_{min}^2 = \frac{3\, \mathrm{ld}(M)}{M - 1}$

Detektion:

$SNR_d \le \frac{E_g}{N_0/2}$

Matched Filter

$H_E(f) = \gamma \cdot G^*(f) e^{-j2\pi f T_d} \circ\!\!-\!\!\bullet\ h_E(t) = \gamma \cdot g^*(T_d - t)$

Nyquist Krit

$\varphi_{gg}(\lambda T) = \int_{-\infty}^{\infty} g(\tau + \lambda T) g^*(\tau) d\tau \stackrel{!}{=} \begin{cases} E_g & \text{für } \lambda = 0 \\ 0 & \text{für } \lambda \in \mathbb{Z} \setminus \{0\} \end{cases}$

$\gamma \cdot a[\ell] E_g \stackrel{!}{=} a[\ell] \Rightarrow \gamma = \frac{1}{E_g}$

$\sum_{\ell=-\infty}^{\infty} |G(f - \ell/T)|^2 = T E_g$

Impuls mit cos-Flanke

$g(t) = \sqrt{\frac{E_g}{T}} \cdot \frac{4\alpha t \cos(\pi(1+\alpha)t/T) + T \sin(\pi(1-\alpha)t/T)}{\pi t(1 - (4\alpha t/T)^2)}$